Proposition 1. *The set of transitions permitted by Kirchhoff's law that Q does not admit consists exactly of the following categories:*

(i) *all transitions $(x, y) \to (x', y')$ where x, y, x', y' all contain a corner point c,*

(ii) *all transitions $(x, y) \to (x', y')$ where x, y, x', y' are all midpoints,*

(iii) *all transitions $(x, y) \to (x', y')$ of type* `dc` $\to$ `dc`*, where $\{x, y\}$ and $\{x', y'\}$ are different double corners, meaning that together with* (i)*, Q does not admit any transitions of type* `dc` $\to$ `dc`*.*

(iv) *all transitions $(x, y) \to (x', y')$ of type* `dc` $\to$ `dm`*, where $\{x, y\}$ and $\{x', y'\}$ do not lie on the same line, meaning that together with* (ii)*, Q does not admit any transitions of type* `dc` $\to$ `dm`*.*

The only transitions permitted by Kirchhoff's law that Q admits are of types `ls` $\to$ `ls`*,* `ls` $\to$ `ang`*,* `alt` $\to$ `alt` *and those transitions $(x, y) \to (x', y')$ of type* `hl` $\to$ `hl`*, where $\{x, y\}$ and $\{x', y'\}$ are different half-lines.*

transition type	representative
`ls` $\to$ `ls`	$\{1, 2\} \to \{1, 2\}$
`ls` $\to$ `ang`	$\{1, 2\} \to \{13, 23\}$
`hl` $\to$ `hl`	$\{1, 12\} \to \{3, 23\}$
`alt` $\to$ `alt`	$\{1, 23\} \to \{1, 23\}$
	$\{1, 23\} \to \{2, 13\}$

Table 1: Transitions of Q up to reversal and premutation of corner points

Proof. First, let us note that by symmetry, for any given pair of shapes `s,t` $\in \Sigma$, it is sufficient to check whether there exist transitions of type `s` $\to$ `t`. Likewise, since there exists a well known automorphism that maps the dangling edges of either connector to one another, given a pair of shapes $\{a, b\}, \{c, d\}$ where a, b, c, d are points of a tetrahedron, it is sufficient to check a single permutation of these points on connectors. Let us prove each of the four cases listed.

(i) Now, we prove that given a corner point c, Q does not admit transitions where each edge receives a value containing c. Assume the contrary. Then, by definition, the edges highlighted in Figure 1 must both receive values containing c, which is impossible since these edges are adjacent.

(ii) Analogously, we prove that Q does not admit any transitions where all dangling edges receive values which are midpoints. Assume the contrary. Then, since each vertex of Q must be adjacent to an edge coloured with a midpoint, the edges highlighted in Figure 1 must both receive values which are midpoints, a contradition.

(iii) Note that there is only one way to assign midpoint values to edges up to the automorphism [Figure 2]. Without loss of generality, let us consider the transition $\{1, 1\} \to \{2, 2\}$ and assign the value 23 to edge ab and the value 4 to edge af. Therefore, since the edge fj cannot receive a value containing 4, it must receive the value 13 and consequently, the edge ij must receive

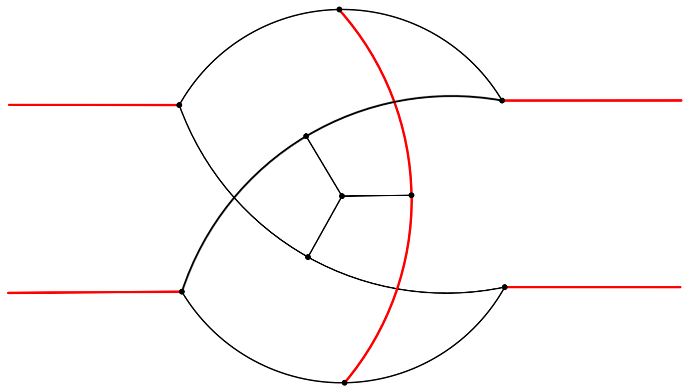

Figure 1: Edges that must receive the contradicting values for cases (i) and (ii) are in red.

the value 4. From that, we can see that the edge hi must receive value 23, the edge dh receives the value 4, the edge cd must receive value 13, and finally, the edge bc must receive value 4. However, that leaves both the edges be and ei to receive the value 1, which is a contradiction.

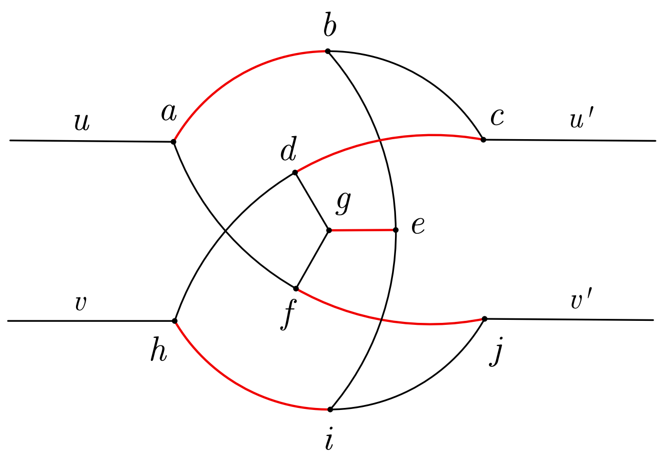

Figure 2: The only way to assign midpoint values to edges for case (iii) up to the automorphism.

(iv) Note that, again, there is only a single way to assign midpoint values to edges up to automorphism [Figure 3]. Without loss of generality, we may take the transition to be $1, 1 \to 23, 23$ and assign the value 23 to edge ab. We now have two cases to consider:

(a) Edge dh receives value 23. In that case, note that both edges af and hi receive value four, meaning that vertex j cannot be incident to an edge which receives value 4, a contradiction.

(b) Edge dh receives value 24 and the edge hi receives value 3. In that case, edges bc and cd must receive values 4 and 1, respectively. Consequently, edge be must receive value 1 and edge dg must receive value 3, meaning that vertex e cannot be incident to an edge which receives value 3, a contradiction.

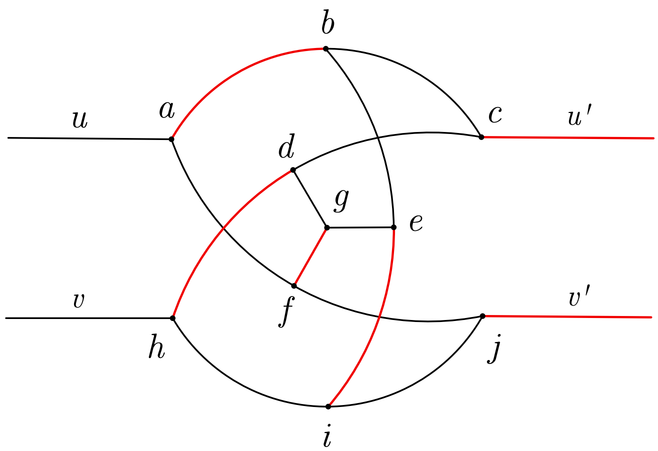

Figure 3: The only way to assign midpoint values to edges for case (iv) up to the automorphism.

Now, we only need to verify that the rest of the transitions exist. Their respective colorings are presented in Figures 4 to 8. □

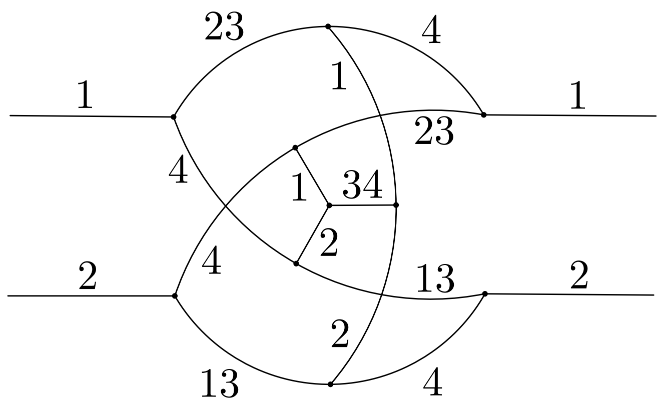

Figure 4: A coloring of Q admitting the transition $\{1, 2\} \to \{1, 2\}$.

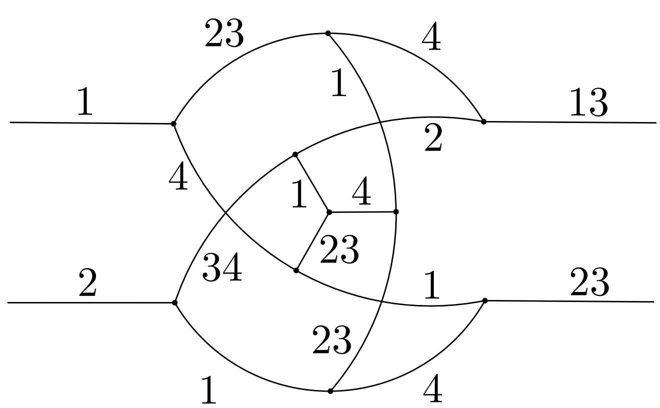

Figure 5: A coloring of Q admitting the transition $\{1, 2\} \to \{13, 23\}$.

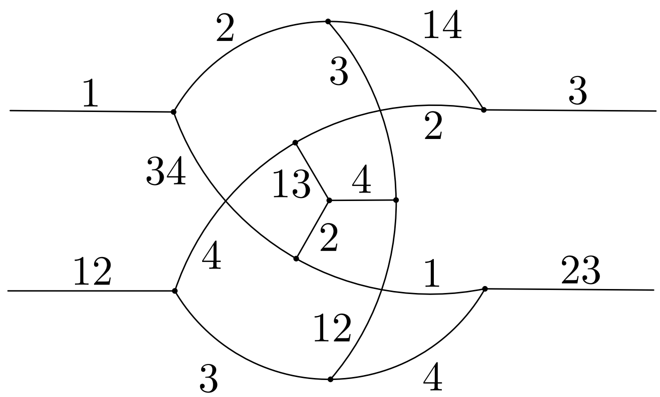

Figure 6: A coloring of Q admitting the transition $\{1, 12\} \to \{3, 23\}$.

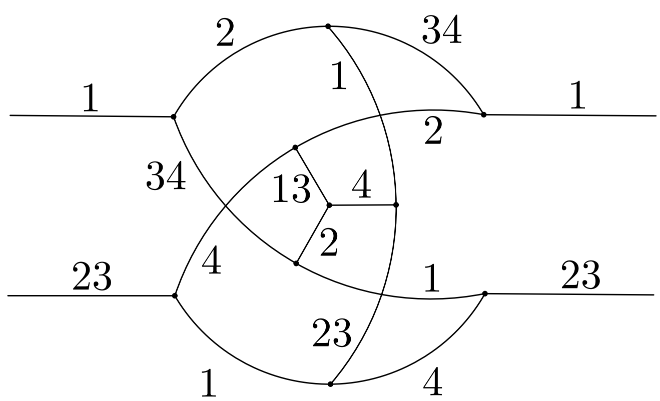

Figure 7: A coloring of Q admitting the transition $\{1, 23\} \to \{1, 23\}$.

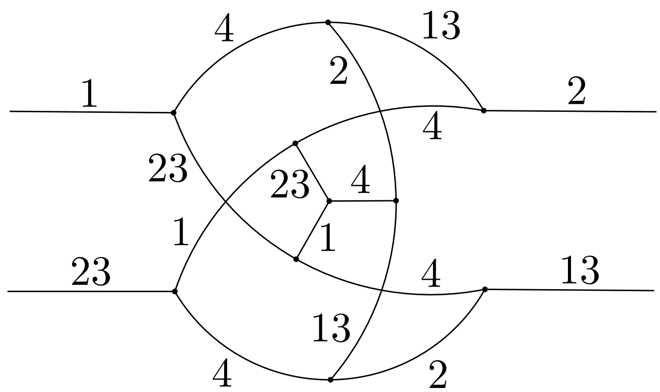

Figure 8: A coloring of Q admitting the transition $\{1, 23\} \to \{2, 13\}$.